MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2016/2017

EMT1026 – ENGINEERING MATHEMATICS II
(All Sections / Groups)

27 FEBRUARY 2017

9.00 AM – 11.00 AM

(2 Hours)

INSTRUCTIONS TO STUDENTS:

1. This exam paper consists of **9 pages** (including cover page) with 4 Questions only.

2. Attempt **all** 4 questions. All questions carry equal marks and the distribution of marks for each question is given.

3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.

4. Several tables are provided in the **Appendix** for your reference – common Laplace transform pairs and properties, special discrete and continuous probability distributions, and the cumulative standard normal distribution.

Question 1

(a)   Find the general solution of the following differential equation:
$$y'' - 6y' + 9y = e^{3x} + 4$$

[12 marks]

(b)   Using power series method, find the first five nonzero terms of the solution for the following differential equation:

$$y'' + 3xy' - 3y = 0$$

[13 marks]

Question 2

(a)   Consider a heat conduction problem as follows:
$$4u_{xx} = u_t, \ 0 \le x \le 3, \ t > 0$$
$$u(0,t) = 0$$
$$u(3,t) = 0$$

Find a general solution for the above heat equation by using the method of separation of variables. Show all the relevant steps.

[18 marks]

(b)   Find the solution for the heat equation in Question 2(a) if the initial condition is $u(x,0) = 10, 0 < x < 3$.

Note: The half-range sine series formulae is

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right), \quad \text{where } b_n = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx.$$

[7 marks]

Continued...

Question 3

(**Note**: The tables in APPENDIX A and APPENDIX B may be useful for solving this question.)

(a)   Let $g(t) = 2t\,e^{-\omega t}$, where ω is a constant. Find the Laplace transform of $g(t)$

        (i)   using the s-Shifting property.   [3 marks]

        (ii) using the Derivative of Transform property.   [3 marks]

(b)   Let $G(s) = \dfrac{2}{s(s^2 + 1)}$. Find the inverse Laplace transform of $G(s)$

        (i)   using the Transform of Integral property.   [3 marks]

        (ii)   using the Convolution Theorem.   [3 marks]

(c)   By using the method of Laplace transforms, solve the following differential equation:

$$y'' + y' - 2y - 1 = 0, \quad y(0) = 0, \quad y'(0) = 6.$$

                                                       [13 marks]

Continued...

Question 4

(a)   The probability mass function, $f(x)$, of a discrete random variable, X, is tabulated below:

x	2	3	4	5	6
$f(x)$	0.3	p	q	r	0.15

(i)   Show that $p + q + r = 0.55$.   [2 marks]

(ii)   Given also that $P(2 < X \le 4) = 0.45$ and $P(4 \le X < 6) = 0.35$. Use this information together with part (i) to deduce that $p = 0.2$, $q = 0.25$ and $r = 0.1$.   [5 marks]

(iii) Find the mean and variance of X.   [5 marks]

(**Note**: For solving Q4(b) and Q4(c) below, you may use the tables in APPENDIX C, APPENDIX D and APPENDIX E.)

(b)   A manufacturer of golf clubs has two machines for producing steel shafts:

- Machine 1 produces shafts with diameters that are normally distributed with mean 30mm and standard deviation 1mm.

- Machine 2 produces shafts with diameters that are normally distributed with mean 30.4mm and standard deviation 0.8mm.

Only shafts that have diameters between 29mm and 31mm are acceptable for export. Which machine is more likely to produce export-quality shafts?

[7 marks]

(c)   Incident reports kept by a manufacturing plant indicate that an average of 3 accidents occur on its premises per week.

(i)   Calculate the probability that 2 or more accidents occur in a given week.

[4 marks]

(ii)   Out of 52 weeks in a year, how many of those weeks would you expect to be accident free?

[2 marks]

End of questions.

APPENDIX A:

Selected Properties of Laplace Transforms

Property Name	Property Relationship
Linearity	$\mathcal{L}\{a\,f(t)+b\,g(t)\}=a\,\mathcal{L}\{f(t)\}+b\,\mathcal{L}\{g(t)\}$
s-Shifting	$\mathcal{L}\{e^{at}\,f(t)\}=F(s-a)$
Transform of Derivative	$\mathcal{L}\{f'\}=s\,\mathcal{L}(f)-f(0)$ $\mathcal{L}\{f''\}=s^2\,\mathcal{L}(f)-s\,f(0)-f'(0)$
Transform of Integral	$\mathcal{L}\{\int_0^t f(\tau)\,d\tau\}=\dfrac{1}{s}F(s)$
Derivative of Transform	$\mathcal{L}\{t\,f(t)\}=-F'(s)$
Integration of Transform	$\mathcal{L}\{\dfrac{f(t)}{t}\}=\int_s^\infty F(\tilde{s})\,d\tilde{s}$
t-shifting	$\mathcal{L}\{u(t-a)f(t-a)\}=e^{-as}F(s)$
Convolution	$\mathcal{L}^{-1}\{F(s)G(s)\}=f(t)*g(t)=\int_0^t f(\tau)\,g(t-\tau)\,d\tau$

APPENDIX B:

Laplace Transforms for Selected *t*-domain Functions

$f(t)$	$F(s) = \mathcal{L}\{f(t)\}$
1	$1/s$
t	$1/s^2$
$t^n \; (n = 1, 2, 3, \ldots)$	$n!/s^{n+1}$
e^{at}	$\dfrac{1}{s-a}$
$t^{n-1}e^{at}$	$\dfrac{(n-1)!}{(s-a)^n}, \; n = 1, 2, \ldots$
$\cos at$	$\dfrac{s}{s^2+a^2}$
$\sin at$	$\dfrac{a}{s^2+a^2}$
$\cosh at$	$\dfrac{s}{s^2-a^2}$
$\sinh at$	$\dfrac{a}{s^2-a^2}$
$u(t-a)$	$\dfrac{e^{-as}}{s}, \; a \geq 0$
$f(t-a)u(t-a)$	$e^{-as}\mathcal{L}(f)$
$f(t)\delta(t-a)$	$e^{-as}f(a)$

Continued…

APPENDIX C:

Special Discrete Probability Distributions

Binomial Distribution, $X \sim b(x; n, p)$	
P.m.f.	$P(X = x) = {}^{n}C_{x} p^{x} q^{n-x}$, for $x = 0, 1, \ldots, n$ and where $q = 1 - p$.
Mean	$E[X] = np$
Variance	$Var(X) = npq$

Hypergeometric Distribution, $X \sim h(x; N, n, k)$	
P.m.f.	$P(X = x) = \dfrac{{}^{k}C_{x} \times {}^{N-k}C_{n-x}}{{}^{N}C_{n}}$, for $x = 0, 1, \ldots \min(n, k)$.
Mean	$E[X] = \dfrac{nk}{N}$
Variance	$Var(X) = \dfrac{nk}{N} \left(\dfrac{N - n}{N - 1} \right) \left(1 - \dfrac{k}{N} \right)$

| Poisson Distribution, $X \sim p(x, \lambda)$	
P.m.f.	$P(X = x) = \dfrac{\lambda^{x} e^{-\lambda}}{x!}$, for $x = 0, 1, \ldots.$
Mean	$E[X] = \lambda$
Variance	$Var(X) = \lambda$

Continued…

APPENDIX D:

Special Continuous Probability Distributions

Continuous Uniform Distribution, $X \sim U(a,b)$	
P.d.f.	$f_X(x) = \begin{cases} \dfrac{1}{b-a}, & a < x < b \\ 0, & \text{elsewhere} \end{cases}$
Mean	$E[X] = \dfrac{a+b}{2}$
Variance	$Var(X) = \dfrac{(b-a)^2}{12}$
Exponential Distribution, $X \sim \exp(1/\beta)$	
P.d.f.	$f_X(x) = \begin{cases} \dfrac{1}{\beta} e^{-x/\beta}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$
Mean	$E[X] = \beta$
Variance	$Var(X) = \beta^2$

Normal Distribution, $X \sim N(\mu, \sigma)$

If X is any normal random variable where $X \sim N(\mu, \sigma)$, then the transformation

$$Z = \frac{X - \mu}{\sigma}$$

yields a *standard* normal variable where $Z \sim N(0,1)$.

Continued...

APPENDIX E:

Cumulative Standard Normal Distribution

$Z \sim N(0,1)$ $F(z) = P(Z \le z)$
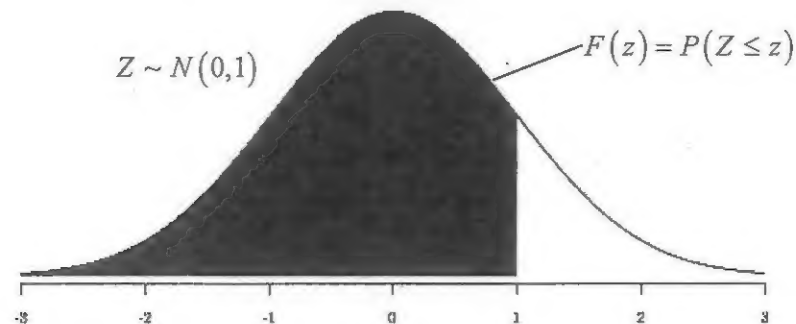
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

End of paper.